華中科技大學
Huazhong University of Science and Technology

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《电力系统分析》（I）

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【教材】
何仰赞，温增银．电力系统分析（上册）（第三版）．武汉：华中科技大学出版社，2002．

华中科技大学
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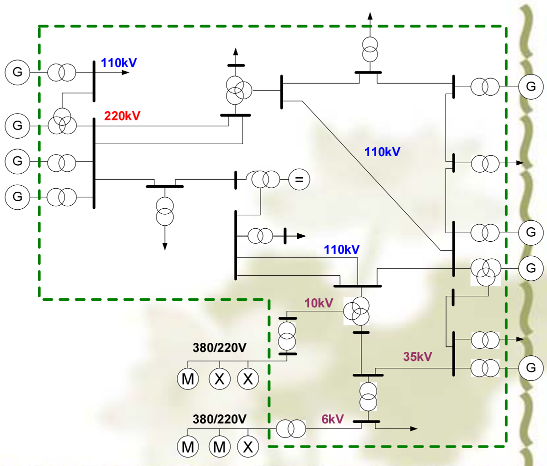
第四章 电力网络的数学模型

通过数学模型，可以把电力系统中物理现象分析归结为某种形式的数学问题。

电力系统数学模型包括：

发电机、负荷、电力网络

本章电力网络数学模型只涉及稳态情况，不考虑网络本身的电磁暂态过程。该模型适合于潮流计算、短路计算、一般稳定计算。



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第四章 电力网络的数学模型

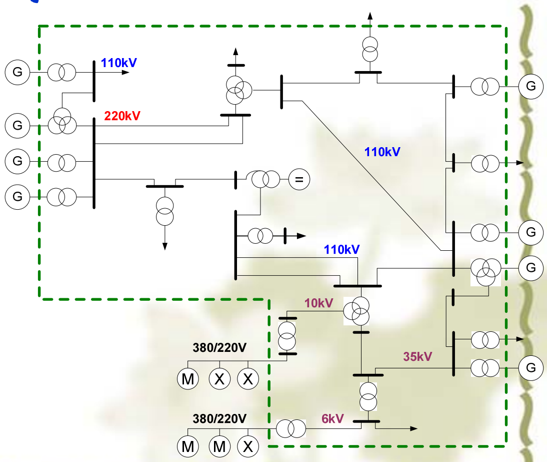
如何实现电力网络的数学抽象？

电力网络物理上就是一系列的元件通过适当的连接而形成的。

两个约束：

1）元件特性，与网络联结无关。U=ZI（无源线性元件）

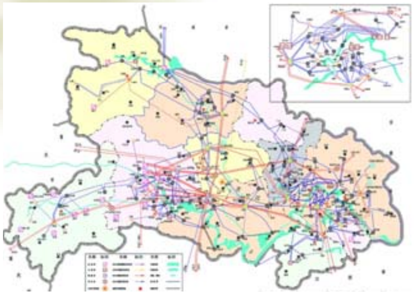
2）网络拓扑，与元件特性无关。KCL、KVL

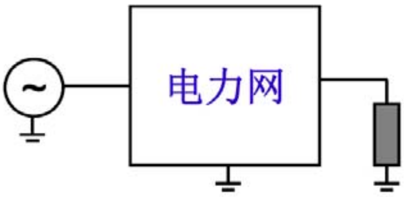


第四章 电力网络的数学模型

4-1 节点导纳矩阵

□ 用于短路计算的电力系统数学模型





- 发电机：电势源支路
- 负荷：恒定阻抗
- 电力网络：网络元件采用一相等值电路，略去变压器励磁支路和线路电容（恒定参数）

电力网络的等值网络成为线性网络。

4-1 节点导纳矩阵

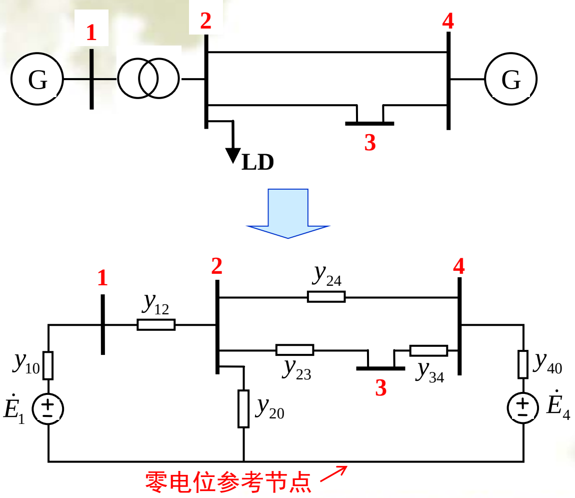
□ 用于短路计算的电力系统数学模型

线性网络求解（n 个独立节点，m 条支路）

- 节点电压法：n 个节点方程，以母线电压为待求量（能唯一地确定网络状态），可直接求出节点电压（便于应用），需要导纳阵或阻抗阵
- 回路电流法：m-n 个回路方程，以回路电流为待求量，需要进一步求各节点电压，需要阻抗阵
- 电力系统计算中一般都采用节点方程！

4-1 节点导纳矩阵

□ 用于短路计算的电力系统数学模型



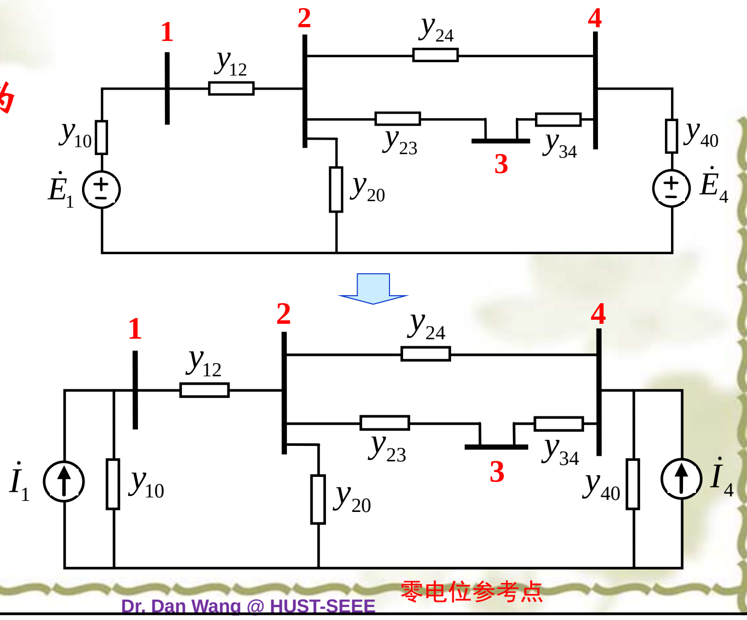
- ❖ 发电机： 电势源支路
- ❖ 负荷： 恒定阻抗
- ❖ 电力网络： 网络元件采用一相等值电路，略去变压器励磁支路和线路电容
- ❖ 零电位参考节点不予编号

4-1 节点导纳矩阵

□ 节点方程

以母线电压为待求量

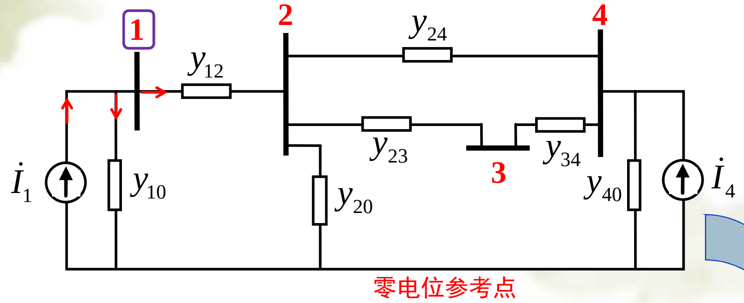
电势源支路转换为电流源支路。



4-1 节点导纳矩阵

□ 节点方程——节点1

以零电位点作为电压参考，根据**KCL定律**列写每一个节点的电流方程式。

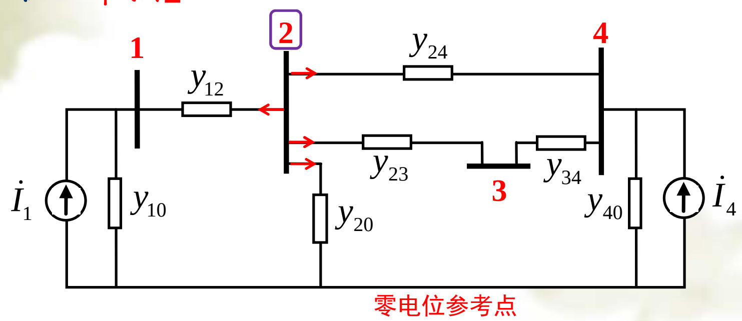


$$\dot{I}_1=(y_{10}+y_{12})\dot{V}_1-y_{12}\dot{V}_2$$

$$\dot{I}_1=y_{10}\dot{V}_1+y_{12}(\dot{V}_1-\dot{V}_2)$$

4-1 节点导纳矩阵

□ 节点方程——节点2

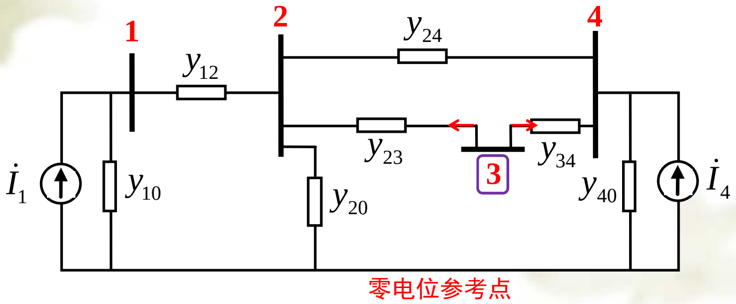


$$0=y_{12}(\dot{V}_2-\dot{V}_1)+y_{20}\dot{V}_2+y_{23}(\dot{V}_2-\dot{V}_3)+y_{24}(\dot{V}_2-\dot{V}_4)$$

$$0=-y_{12}\dot{V}_1+(y_{12}+y_{20}+y_{23}+y_{24})\dot{V}_2-y_{23}\dot{V}_3-y_{24}\dot{V}_4$$

4-1 节点导纳矩阵

□ 节点方程——节点3

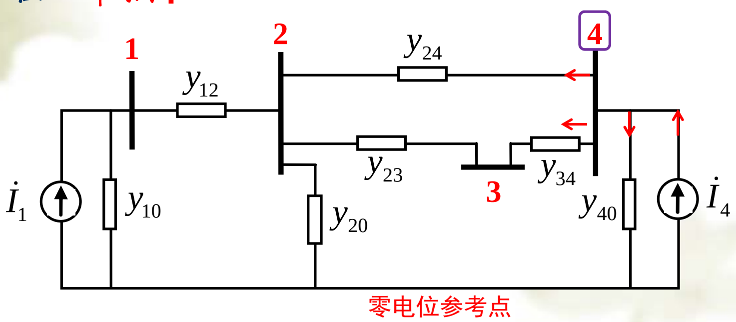


$$0 = y_{23}(\dot{V}_3 - \dot{V}_2) + y_{34}(\dot{V}_3 - \dot{V}_4)$$

$$0 = -y_{23}\dot{V}_2 + \left(y_{23} + y_{34}\right)\dot{V}_3 - y_{34}\dot{V}_4$$

4-1 节点导纳矩阵

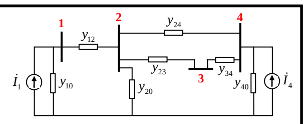
□ 节点方程——节点4



$$\dot{I}_4 = y_{24}(\dot{V}_4 - \dot{V}_2) + y_{34}(\dot{V}_4 - \dot{V}_3) + y_{40}\dot{V}_4$$

$$\dot{I}_4 = -y_{24}\dot{V}_2 - y_{34}\dot{V}_3 + \left(y_{24} + y_{34} + y_{40}\right)\dot{V}_4$$

4-1 节点导纳矩阵



□ 节点方程的矩阵形式

$$\begin{cases}\dot{I}_1=(y_{10}+y_{12})\dot{V}_1-y_{12}\dot{V}_2\\ 0=-y_{12}\dot{V}_1+(y_{12}+y_{20}+y_{23}+y_{24})\dot{V}_2-y_{23}\dot{V}_3-y_{24}\dot{V}_4\\ 0=-y_{23}\dot{V}_3+(y_{23}+y_{34})\dot{V}_3-y_{34}\dot{V}_4\\ \dot{I}_4=-y_{24}\dot{V}_2-y_{34}\dot{V}_3+(y_{24}+y_{34}+y_{40})\dot{V}_4\end{cases}$$

$$Y_{12}=Y_{21}=-y_{12}$$
$$Y_{23}=Y_{32}=-y_{23}$$
$$Y_{24}=Y_{42}=-y_{24}$$
$$Y_{34}=Y_{43}=-y_{34}$$

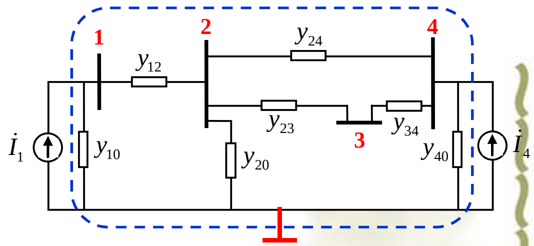
$$\begin{bmatrix}\dot{I}_1\\0\\0\\\dot{I}_4\end{bmatrix}=\begin{bmatrix}Y_{11}&Y_{12}&0&0\\Y_{21}&Y_{22}&Y_{23}&Y_{24}\\0&Y_{32}&Y_{33}&Y_{34}\\0&Y_{42}&Y_{43}&Y_{44}\end{bmatrix}\begin{bmatrix}\dot{V}_1\\\dot{V}_2\\\dot{V}_3\\\dot{V}_4\end{bmatrix}$$

$$Y_{11}=y_{10}+y_{12}$$
$$Y_{22}=y_{12}+y_{20}+y_{23}+y_{24}$$
$$Y_{33}=y_{23}+y_{34}$$
$$Y_{44}=y_{24}+y_{34}+y_{40}$$

4-1 节点导纳矩阵

□ 节点方程的矩阵形式

$$\begin{bmatrix}\dot{I}_1\\0\\0\\\dot{I}_4\end{bmatrix}=\begin{bmatrix}Y_{11}&Y_{12}&0&0\\Y_{21}&Y_{22}&Y_{23}&Y_{24}\\0&Y_{32}&Y_{33}&Y_{34}\\0&Y_{42}&Y_{43}&Y_{44}\end{bmatrix}\begin{bmatrix}\dot{V}_1\\\dot{V}_2\\\dot{V}_3\\\dot{V}_4\end{bmatrix}$$



$$\begin{bmatrix}\dot{I}_1\\\dot{I}_2\\\dot{I}_3\\\dot{I}_4\end{bmatrix}=\begin{bmatrix}Y_{11}&Y_{12}&Y_{13}&Y_{14}\\Y_{21}&Y_{22}&Y_{23}&Y_{24}\\Y_{31}&Y_{32}&Y_{33}&Y_{34}\\Y_{34}&Y_{42}&Y_{43}&Y_{44}\end{bmatrix}\begin{bmatrix}\dot{V}_1\\\dot{V}_2\\\dot{V}_3\\\dot{V}_4\end{bmatrix}$$

节点注入电流：$\dot{\mathbf{I}}=\begin{bmatrix}\dot{I}_1&\dot{I}_2&\dot{I}_3&\dot{I}_4\end{bmatrix}^{\mathrm{T}}$

节点电压：$\dot{\mathbf{V}}=\begin{bmatrix}\dot{V}_1&\dot{V}_2&\dot{V}_3&\dot{V}_4\end{bmatrix}^{\mathrm{T}}$

$$\dot{\mathbf{I}}=\mathbf{Y}\dot{\mathbf{V}}$$

4-1 节点导纳矩阵

□ 节点方程——n个独立节点的电力网络数学模型

$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \vdots \\ \dot{I}_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \vdots \\ \dot{V}_n \end{bmatrix}$$

线性代数方程

$$\dot{\mathbf{I}} = \mathbf{Y}\dot{\mathbf{V}}$$

Y_{ii} 节点i自导纳

Y_{ij} 节点ij间互导纳

节点导纳矩阵 **Y**

4-1 节点导纳矩阵

$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \vdots \\ \dot{I}_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \vdots \\ \dot{V}_n \end{bmatrix}$$

□ 节点导纳矩阵元素的物理意义

$$\dot{\mathbf{I}} = \mathbf{Y}\dot{\mathbf{V}}$$

$$\begin{cases} \dot{I}_i = \sum_{k=1}^{n} Y_{ik}\dot{V}_k \\ \dot{I}_j = \sum_{k=1}^{n} Y_{jk}\dot{V}_k \end{cases}$$

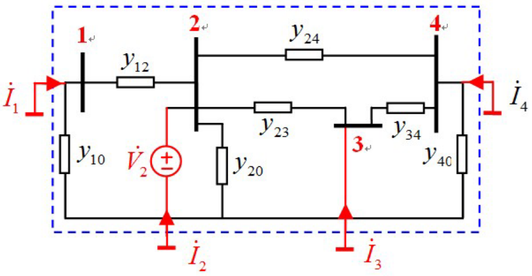
$\dot{V}_i \neq 0,$

$\dot{V}_k = 0,$

$(k \neq i)$

$$\dot{I}_i = Y_{ii}\dot{V}_i \Rightarrow Y_{ii} = \dot{I}_i / \dot{V}_i$$

$$\dot{I}_j = Y_{ji}\dot{V}_i \Rightarrow Y_{ji} = \dot{I}_j / \dot{V}_i$$



$$Y_{22} = \dot{I}_2 / \dot{V}_2 = y_{20} + y_{12} + y_{23} + y_{24}$$

$$Y_{12} = \dot{I}_1 / \dot{V}_2 = -y_{12}, \quad Y_{32} = \dot{I}_3 / \dot{V}_2 = -y_{23}$$

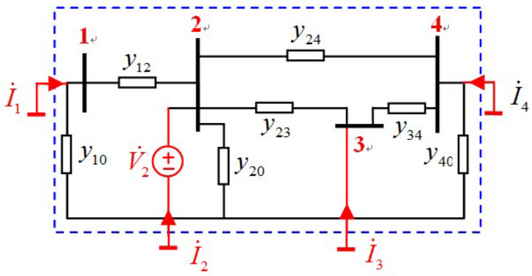
$$Y_{42} = \dot{I}_4 / \dot{V}_2 = -y_{24}$$

4-1 节点导纳矩阵

$$\begin{bmatrix}\dot I_1\\ \dot I_2\\ \vdots\\ \dot I_n\end{bmatrix}=\begin{bmatrix}Y_{11} & Y_{12} & \cdots & Y_{1n}\\ Y_{21} & Y_{22} & \cdots & Y_{2n}\\ \vdots & \vdots & & \vdots\\ Y_{n1} & Y_{n2} & \cdots & Y_{nn}\end{bmatrix}\begin{bmatrix}\dot V_1\\ \dot V_2\\ \vdots\\ \dot V_n\end{bmatrix}$$

□ 节点导纳矩阵元素的物理意义——自导纳

Y_{ii}：当网络中除节点i以外所有节点都接地时，从节点i注入网络的电流同施加于节点i的 电压之比。



Y_{ii} 等于与节点i相连的所有支路导纳之和。

$$\text{if } i=j \Rightarrow Y_{ii}=\dot I_i/\dot V_i$$

$$Y_{ii}=y_{i0}+\sum_j y_{ij}$$

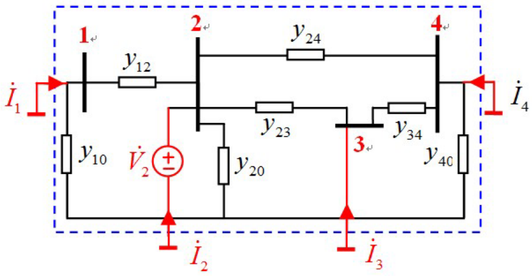
$$Y_{22}=\dot I_2/\dot V_2=y_{20}+y_{12}+y_{23}+y_{24}$$

4-1 节点导纳矩阵

$$\begin{bmatrix}\dot I_1\\ \dot I_2\\ \vdots\\ \dot I_n\end{bmatrix}=\begin{bmatrix}Y_{11} & Y_{12} & \cdots & Y_{1n}\\ Y_{21} & Y_{22} & \cdots & Y_{2n}\\ \vdots & \vdots & & \vdots\\ Y_{n1} & Y_{n2} & \cdots & Y_{nn}\end{bmatrix}\begin{bmatrix}\dot V_1\\ \dot V_2\\ \vdots\\ \dot V_n\end{bmatrix}$$

□ 节点导纳矩阵元素的物理意义——互导纳

Y_{ji}：当网络中除节点i以外所有节点都接地时，从节点j注入网络的电流同施加于节点i的电压之比。

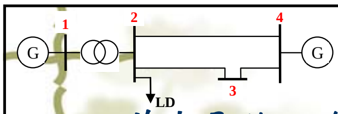


节点j的电流实际上是自网络流出并进入地中的电流，所以Y_{ji}应等于节点j、i之间连接支路导纳的负值。

$$\text{if } i\neq j \quad Y_{ij}=\dot I_i/\dot V_j$$

$$Y_{ij}=-y_{ij}$$

$$Y_{12}=\dot I_1/\dot V_2=-y_{12}$$
$$Y_{32}=\dot I_3/\dot V_2=-y_{23}$$
$$Y_{42}=\dot I_4/\dot V_2=-y_{24}$$



4-1 节点导纳矩阵

$$\begin{bmatrix}\dot I_1\\0\\0\\\dot I_4\end{bmatrix}=\begin{bmatrix}Y_{11}&Y_{12}&0&0\\Y_{21}&Y_{22}&Y_{23}&Y_{24}\\0&Y_{32}&Y_{33}&Y_{34}\\0&Y_{42}&Y_{43}&Y_{44}\end{bmatrix}\begin{bmatrix}\dot V_1\\\dot V_2\\\dot V_3\\\dot V_4\end{bmatrix}$$

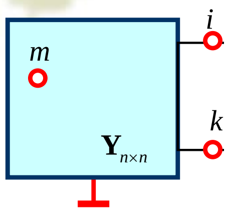
节点导纳矩阵特点

- 直观易得 $\Leftarrow$ $Y_{ii}=y_{i0}+\sum y_{ij}$，$Y_{ij}=-\sum y_{ij}$
- 对称矩阵 $\Leftarrow$ $Y_{ij}=Y_{ji}=-\sum y_{ij}$
- 稀疏矩阵 $\Leftarrow$ 如果*ij*之间没有直接支路连接，则：$Y_{ij}=Y_{ji}=0$

$\mathbf{Y}_{n\times n}$ 电力系统一般每个节点平均有3~4条出线，节点导纳矩阵每行非零元素非常少！

4-1 节点导纳矩阵

Y阵的修改



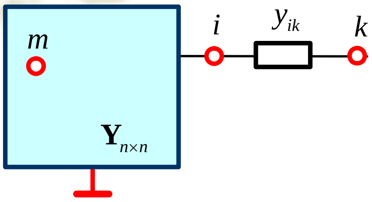
电力系统运行状态会不断变化，网络结构会根据运行要求改变，网络结构改变时，节点导纳阵也要作相应修改。

$$Y_{ij}=Y_{ij}^{(0)}+\Delta Y_{ij}$$

技巧：利用导纳阵元素的物理意义。

4-1 节点导纳矩阵

□ Y阵的修改——增加树支（增加一个节点）



$$\begin{bmatrix} Y_{11} & \cdots & Y_{1i} & \cdots & Y_{1n} & 0 \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Y_{i1} & \cdots & Y_{ii} & \cdots & Y_{in} & Y_{ik} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Y_{n1} & \cdots & Y_{ni} & \cdots & Y_{nn} & 0 \\ 0 & \cdots & Y_{ki} & \cdots & 0 & Y_{kk} \end{bmatrix}$$

$$Y_{ii} = Y_{ii}^{(0)} + y_{ik}$$
$$Y_{kk} = y_{ik}$$
$$Y_{ik} = Y_{ki} = -y_{ik}$$

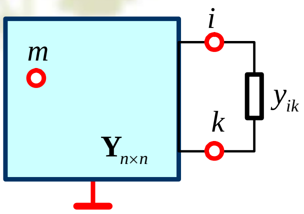
$$Y_{mk} = Y_{km} = 0$$
$$Y_{mi} = Y_{im} = Y_{im}^{(0)}$$

Y阵增加一行一列

$\dot{I}_i = Y_{ii}\dot{V}_i \quad \dot{I}_j = Y_{ji}\dot{V}_i$

4-1 节点导纳矩阵

□ Y阵的修改——增加连支（增加一条支路）



$$\begin{bmatrix} Y_{11} & \cdots & Y_{1i} & \cdots & Y_{1k} & \cdots & Y_{1n} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ Y_{i1} & \cdots & Y_{ii} & \cdots & Y_{ik} & \cdots & Y_{in} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ Y_{k1} & \cdots & Y_{ki} & \cdots & Y_{kk} & \cdots & Y_{kn} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ Y_{n1} & \cdots & Y_{ni} & v & Y_{nk} & \cdots & Y_{nn} \end{bmatrix}$$

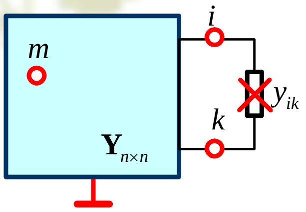
$$Y_{ii} = Y_{ii}^{(0)} + y_{ik}$$
$$Y_{kk} = Y_{kk}^{(0)} + y_{ik}$$
$$Y_{ik} = Y_{ki} = Y_{ik}^{(0)} - y_{ik}$$

Y阵阶数保持不变

$$Y_{mk} = Y_{km} = Y_{mk}^{(0)}, \quad Y_{mi} = Y_{im} = Y_{im}^{(0)}$$

4-1 节点导纳矩阵

Y阵的修改——删除连支（切除一条支路）



$$\begin{bmatrix} Y_{11} & \cdots & Y_{1i} & \cdots & Y_{1k} & \cdots & Y_{1n} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ Y_{i1} & \cdots & Y_{ii} & \cdots & Y_{ik} & \cdots & Y_{in} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ Y_{k1} & \cdots & Y_{ki} & \cdots & Y_{kk} & \cdots & Y_{kn} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ Y_{n1} & \cdots & Y_{ni} & \cdots & Y_{nk} & \cdots & Y_{nn} \end{bmatrix}$$

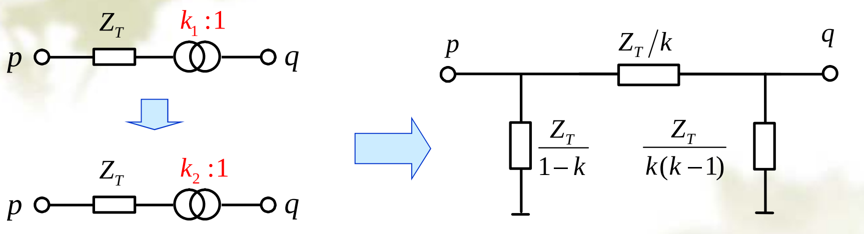
$$Y_{ii} = Y_{ii}^{(0)} - y_{ik}$$
$$Y_{kk} = Y_{kk}^{(0)} - y_{ik}$$
$$Y_{ik} = Y_{ki} = Y_{ik}^{(0)} + y_{ik}$$

Y阵阶数保持不变

$$Y_{mk} = Y_{km} = Y_{mk}^{(0)}, \quad Y_{mi} = Y_{im} = Y_{im}^{(0)}$$

4-1 节点导纳矩阵

Y阵的修改——改变变压器变比



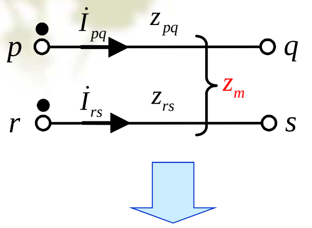
$$Y_{pp} = Y_{pp}^{(0)} - \Delta Y_{pp}^{(1)} + \Delta Y_{pp}^{(2)}$$
$$Y_{qq} = Y_{qq}^{(0)} - \Delta Y_{qq}^{(1)} + \Delta Y_{qq}^{(2)}$$
$$Y_{pq} = Y_{qp} = Y_{pq}^{(0)} - \Delta Y_{pq}^{(1)} + \Delta Y_{pq}^{(2)}$$

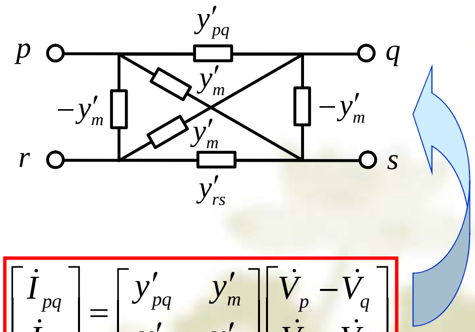
$$\Delta Y_{pp} = k/Z_T + (1-k)/Z_T = 1/Z_T$$
$$\Delta Y_{qq} = k/Z_T + k(k-1)/Z_T = k^2/Z_T$$
$$\Delta Y_{pq} = \Delta Y_{qp} = -k/Z_T$$

自行分析例4-2

4-1 节点导纳矩阵

□ Y阵的修改——支路间存在互感



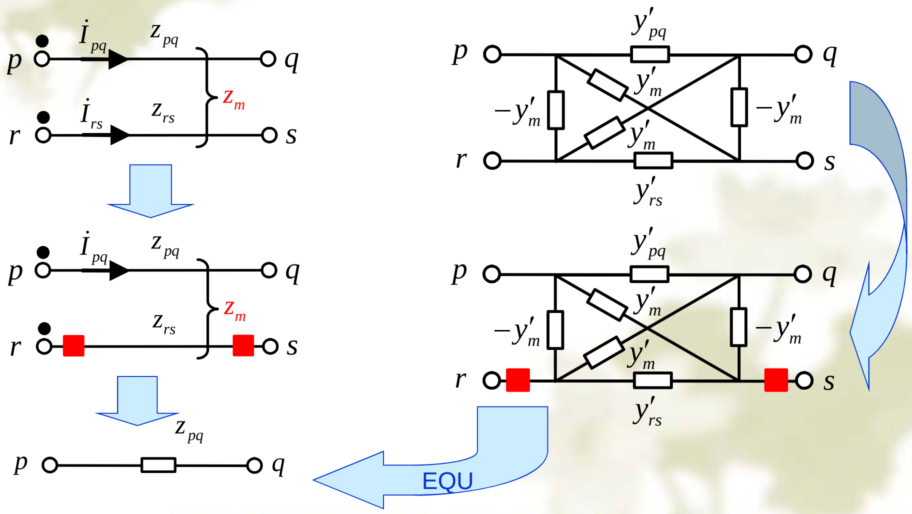


$$\begin{bmatrix} \dot{V}_p - \dot{V}_q \\ \dot{V}_r - \dot{V}_s \end{bmatrix} = \begin{bmatrix} z_{pq} & z_m \\ z_m & z_{rs} \end{bmatrix} \begin{bmatrix} \dot{I}_{pq} \\ \dot{I}_{rs} \end{bmatrix}$$

$$\begin{bmatrix} \dot{I}_{pq} \\ \dot{I}_{rs} \end{bmatrix} = \begin{bmatrix} y'_{pq} & y'_m \\ y'_m & y'_{rs} \end{bmatrix} \begin{bmatrix} \dot{V}_p - \dot{V}_q \\ \dot{V}_r - \dot{V}_s \end{bmatrix}$$

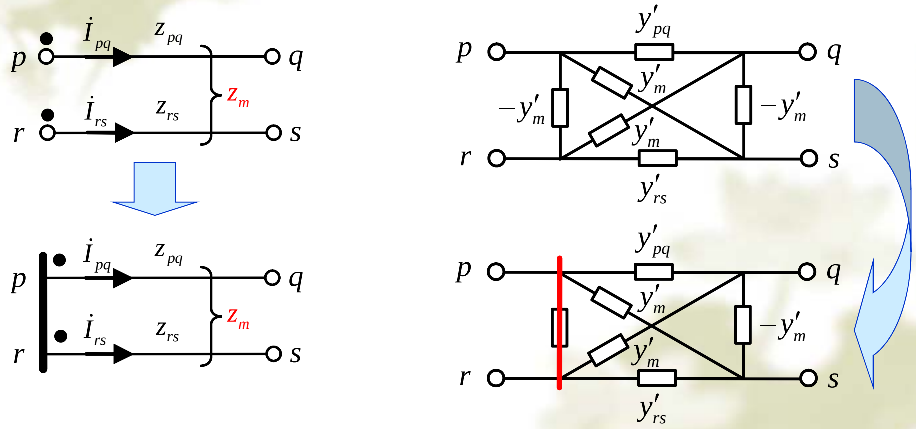
4-1 节点导纳矩阵

□ Y阵的修改——断开互感支路



4-1 节点导纳矩阵

□ Y阵的修改——一端互联的互感支路



4-2 网络方程的解法

□ 高斯消去法

导纳型节点方程采用高斯消去法，实际上就是带有节点电流移置的星网变换（物理意义）。

（自学！）

4-3 节点阻抗矩阵

□ 节点阻抗矩阵

$$\begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \vdots \\ \dot{V}_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \vdots \\ \dot{I}_n \end{bmatrix}$$

$$\dot{\mathbf{I}} = \mathbf{Y}\dot{\mathbf{V}}$$

$$\dot{\mathbf{V}} = \mathbf{Z}\dot{\mathbf{I}}$$

Z_{ii} 节点*i*自阻抗

Z_{ij} 节点*ij*间互阻抗

节点阻抗矩阵

$$\mathbf{Z} = \mathbf{Y}^{-1}$$

4-3 节点阻抗矩阵

$$\begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \vdots \\ \dot{V}_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \vdots \\ \dot{I}_n \end{bmatrix}$$

□ 节点阻抗矩阵元素的物理意义

$$\dot{\mathbf{V}} = \mathbf{Z}\dot{\mathbf{I}}$$

$$\begin{cases} \dot{V}_i = \sum_{k=1}^{n} Z_{ik}\dot{I}_k \\ \dot{V}_j = \sum_{k=1}^{n} Z_{jk}\dot{I}_k \end{cases}$$

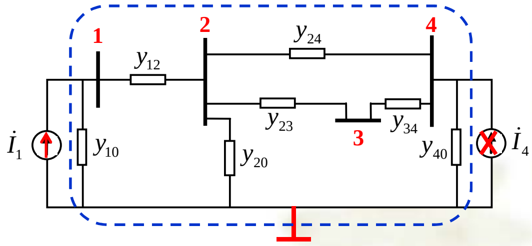
$\dot{I}_i \neq 0,$

$\dot{I}_j = 0,$

$(j \neq i)$

$$\dot{V}_i = Z_{ii}\dot{I}_i \Rightarrow Z_{ii} = \dot{V}_i / \dot{I}_i$$

$$\dot{V}_j = Z_{ji}\dot{I}_i \Rightarrow Z_{ji} = \dot{V}_j / \dot{I}_i$$



当在节点 *i* 单独注入电流，而所有其他节点的注入电流都等于零时，在节点 *i* 产生的电压同注入电流之比，即是节点 *i* 的自阻抗 Z_{ii}。

4-3 节点阻抗矩阵

$$\begin{bmatrix}\dot V_1\\ \dot V_2\\ \vdots\\ \dot V_n\end{bmatrix}=\begin{bmatrix}Z_{11} & Z_{12} & \cdots & Z_{1n}\\ Z_{21} & Z_{22} & \cdots & Z_{2n}\\ \vdots & \vdots & \cdots & \vdots\\ Z_{n1} & Z_{n2} & \cdots & Z_{nn}\end{bmatrix}\begin{bmatrix}\dot I_1\\ \dot I_2\\ \vdots\\ \dot I_n\end{bmatrix}$$

□ 节点阻抗矩阵元素的物理意义

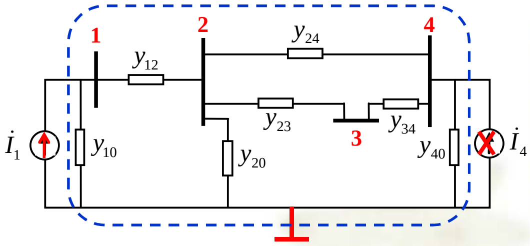
$$\dot{\mathbf{V}}=\mathbf{Z}\dot{\mathbf{I}}$$

$$\begin{cases}\dot V_i=\sum_{k=1}^{n} Z_{ik}\dot I_k\\ \dot V_j=\sum_{k=1}^{n} Z_{jk}\dot I_k\end{cases}$$

$\dot I_i\neq 0,$ $\dot I_j=0,$ $(j\neq i)$

$$\dot V_i=Z_{ii}\dot I_i\Rightarrow Z_{ii}=\dot V_i/\dot I_i$$

$$\dot V_j=Z_{ji}\dot I_i\Rightarrow Z_{ji}=\dot V_j/\dot I_i$$



当在节点 *i* 单独注入电流，而所有其他节点的注入电流都等于零时，在节点 *j* 产生的电压同节点 *i* 注入电流之比，即是节点 *i* 和节点 *j* 之间的互阻抗 Z_{ij}。

4-3 节点阻抗矩阵

□ 节点阻抗矩阵元素的物理意义

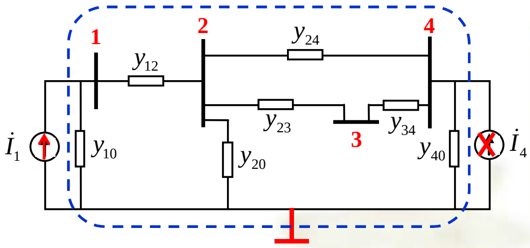
$$\dot{\mathbf{V}}=\mathbf{Z}\dot{\mathbf{I}}$$

$$\begin{cases}\dot V_i=\sum_{k=1}^{n} Z_{ik}\dot I_k\\ \dot V_j=\sum_{k=1}^{n} Z_{jk}\dot I_k\end{cases}$$

$\dot I_i\neq 0,$ $\dot I_j=0,$ $(j\neq i)$

$$\dot V_i=Z_{ii}\dot I_i\Rightarrow Z_{ii}=\dot V_i/\dot I_i$$

$$\dot V_j=Z_{ji}\dot I_i\Rightarrow Z_{ji}=\dot V_j/\dot I_i$$



特点：

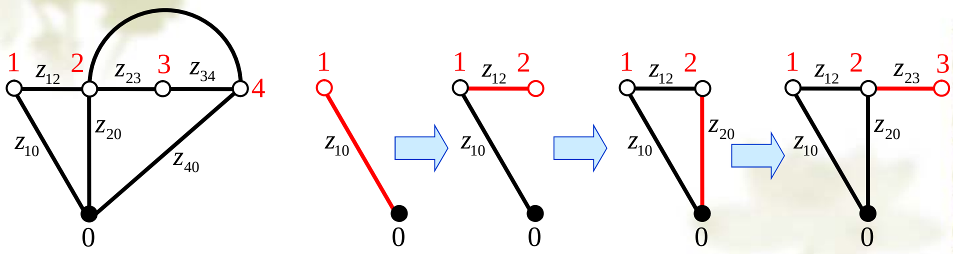
❖对称矩阵　　❖满阵

❖计算复杂

求取方法：1) 支路追加法，或2) 导纳矩阵求逆

4-3 节点阻抗矩阵

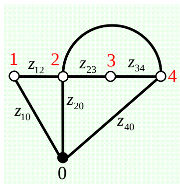
□ 支路追加法生成Z阵-基本过程



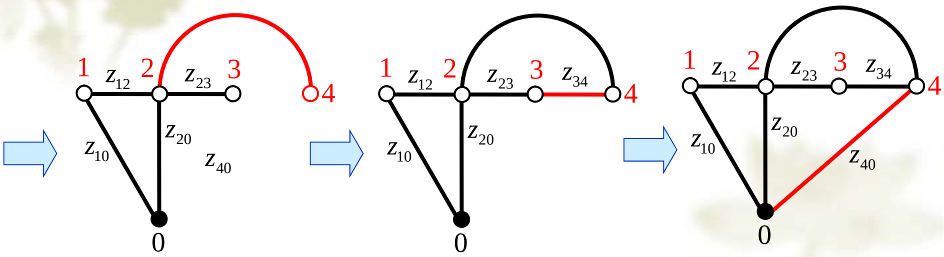
$\mathbf{Z}_{4\times4}$ $\mathbf{Z}_{1\times1}$ $\mathbf{Z}_{2\times2}$ $\mathbf{Z}'_{2\times2}$ $\mathbf{Z}_{3\times3}$

原则：1、从某一个与地相连的支路开始，逐步增加支路；
2、追加的支路至少有一个端点同已出现的节点相接。

4-3 节点阻抗矩阵



□ 支路追加法生成Z阵-基本过程

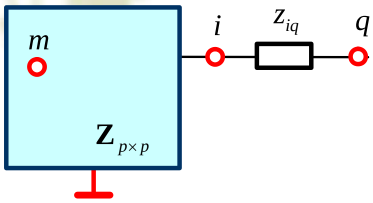


$\mathbf{Z}'_{4\times4}$ $\mathbf{Z}''_{4\times4}$ $\mathbf{Z}_{4\times4}$

追加的支路要么是树支（引入新节点），要么是连支。

4-3 节点阻抗矩阵

- □ 支路追加法生成Z阵-追加树支



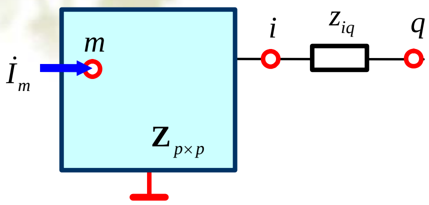
$$\left[\begin{array}{ccccc|c} Z_{11} & \cdots & Z_{1i} & \cdots & Z_{1p} & Z_{1q} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{i1} & \cdots & Z_{ii} & \cdots & Z_{ip} & Z_{iq} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pi} & \cdots & Z_{pp} & Z_{pq} \\ \hline Z_{q1} & \cdots & Z_{qi} & \cdots & Z_{qp} & Z_{qq} \end{array}\right]$$

$\mathbf{Z}_{p\times p}$ ⇒ $\mathbf{Z}_{(p+1)\times(p+1)}$

Z阵增加一行一列

4-3 节点阻抗矩阵

- □ 支路追加法生成Z阵-追加树支



$$\left[\begin{array}{ccccc|c} Z_{11} & \cdots & Z_{1i} & \cdots & Z_{1p} & Z_{1q} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{i1} & \cdots & Z_{ii} & \cdots & Z_{ip} & Z_{iq} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pi} & \cdots & Z_{pp} & Z_{pq} \\ \hline Z_{q1} & \cdots & Z_{qi} & \cdots & Z_{qp} & Z_{qq} \end{array}\right]$$

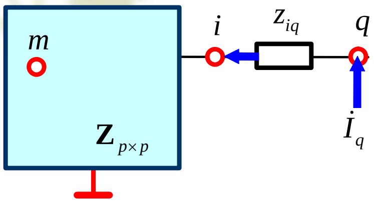
m节点单独注入电流$\dot{I}_m$

$\dot{V}_i = Z_{im}\dot{I}_m$

$\dot{V}_q = Z_{qm}\dot{I}_m = \dot{V}_i = Z_{im}\dot{I}_m$ ⇒ $Z_{qm} = Z_{im}$

4-3 节点阻抗矩阵

- 支路追加法生成Z阵-追加树支



$$\begin{bmatrix} Z_{11} & \cdots & Z_{1i} & \cdots & Z_{1p} & Z_{1q} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{i1} & \cdots & Z_{ii} & \cdots & Z_{ip} & Z_{iq} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pi} & \cdots & Z_{pp} & Z_{pq} \\ Z_{q1} & \cdots & Z_{qi} & \cdots & Z_{qp} & Z_{qq} \end{bmatrix}$$

q节点单独注入电流$\dot{I}_q$

$\dot{V}_m = Z_{mq}\dot{I}_q = Z_{mi}\dot{I}_q$

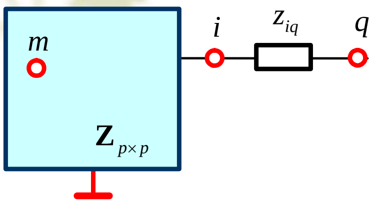
$Z_{mq} = Z_{mi}$

$\dot{V}_q = \dot{V}_i + z_{iq}\dot{I}_q = \left(Z_{ii} + z_{iq}\right)\dot{I}_q = Z_{qq}\dot{I}_q$

$Z_{qq} = Z_{ii} + z_{iq}$

4-3 节点阻抗矩阵

- 支路追加法生成Z阵-追加树支



$$\begin{bmatrix} Z_{11} & \cdots & Z_{1i} & \cdots & Z_{1p} & Z_{1q} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{i1} & \cdots & Z_{ii} & \cdots & Z_{ip} & Z_{iq} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pi} & \cdots & Z_{pp} & Z_{pq} \\ Z_{q1} & \cdots & Z_{qi} & \cdots & Z_{qp} & Z_{qq} \end{bmatrix}$$

Z阵增加一行一列

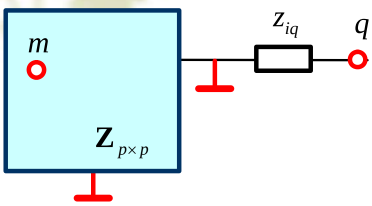
Z阵原有元素不变

$Z_{mq} = Z_{qm} = Z_{mi}, \left(m = 1, 2, \cdots, p\right)$

$Z_{qq} = Z_{ii} + z_{iq}$

4-3 节点阻抗矩阵

□ 支路追加法生成Z阵-追加接地树支



$$\begin{bmatrix} Z_{11} & \cdots & Z_{1i} & \cdots & Z_{1p} & Z_{1q} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{i1} & \cdots & Z_{ii} & \cdots & Z_{ip} & Z_{iq} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pi} & \cdots & Z_{pp} & Z_{pq} \\ Z_{q1} & \cdots & Z_{qi} & \cdots & Z_{qp} & Z_{qq} \end{bmatrix}$$

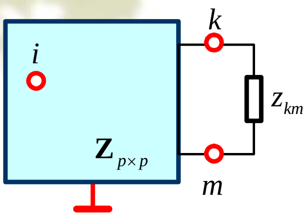
Z阵增加一行一列

Z阵原有元素不变

$Z_{mq} = Z_{qm} = 0, (m = 1, 2, \cdots, p)$ $Z_{qq} = z_{iq}$

4-3 节点阻抗矩阵

□ 支路追加法生成Z阵-追加连支



$Z_{p\times p}$ ⇒ $Z'_{p\times p}$

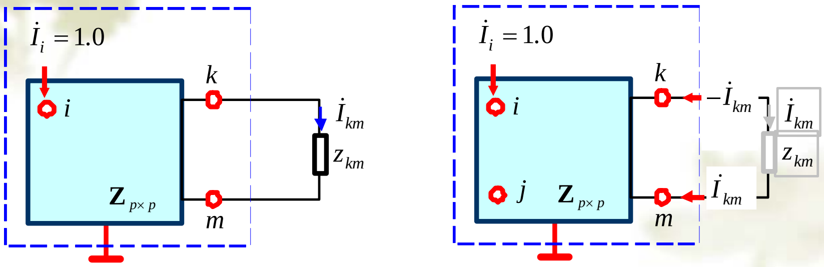
$$\begin{bmatrix} Z'_{11} & \cdots & Z'_{1m} & \cdots & Z'_{1k} & \cdots \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z'_{m1} & \cdots & Z'_{mm} & \cdots & Z'_{mk} & \cdots \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z'_{k1} & \cdots & Z'_{km} & \cdots & Z'_{kk} & \cdots \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \end{bmatrix}$$

Z阵阶数保持不变

z_{km}支路会引起原网络电压电流分布的变化

4-3 节点阻抗矩阵

□ 支路追加法生成Z阵-追加连支（方法I）



$$\dot{V}_j = Z'_{ij}\dot{I}_i = Z_{ij}\dot{I}_i - Z_{kj}\dot{I}_{km} + Z_{mj}\dot{I}_{km} = Z_{ij}\dot{I}_i - \left(Z_{kj} - Z_{mj}\right)\dot{I}_{km}$$

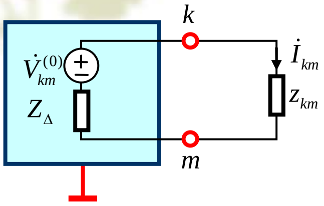
$$\dot{V}_j = Z_{ij} - \left(Z_{kj} - Z_{mj}\right)\dot{I}_{km}$$

关键：$\dot{I}_{km} = ?$

注：此处假定只有节点i单独有单位电流注入。

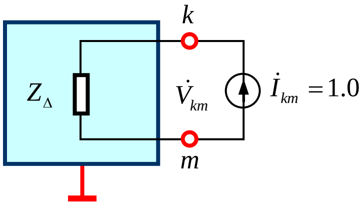
4-3 节点阻抗矩阵

□ 支路追加法生成Z阵-追加连支（方法I）



$$\dot{I}_{km} = \frac{\dot{V}_{km}^{(0)}}{Z_{\Delta} + z_{km}} = \frac{\dot{V}_k^{(0)} - \dot{V}_m^{(0)}}{Z_{\Delta} + z_{km}} = \frac{Z_{ik} - Z_{im}}{Z_{\Delta} + z_{km}}$$

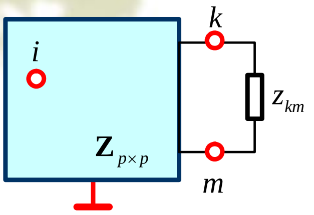
关键：$Z_{\Delta} = ?$



$$Z_{\Delta} = \left(Z_{kk} - Z_{km}\right) - \left(Z_{km} - Z_{mm}\right) = Z_{kk} + Z_{mm} - 2Z_{km}$$

4-3 节点阻抗矩阵

- 支路追加法生成Z阵-追加连支（方法I）



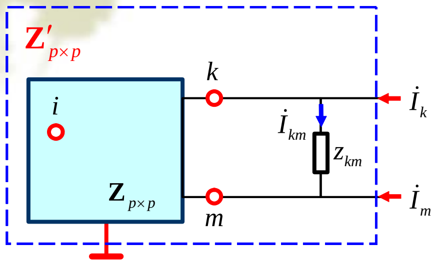
$$Z'_{ij}=\frac{\dot V_j}{\dot I_i}=Z_{ij}-\left(Z_{kj}-Z_{mj}\right)\dot I_{km}$$

$$=Z_{ij}-\frac{\left(Z_{ik}-Z_{im}\right)\left(Z_{kj}-Z_{mj}\right)}{\left(Z_{kk}+Z_{mm}-2Z_{km}+z_{km}\right)}\qquad (i,j=1,2,\cdots,p)$$

4-3 节点阻抗矩阵

$$\begin{bmatrix}\dot V_1\\ \dot V_2\\ \vdots\\ \dot V_n\end{bmatrix}=\begin{bmatrix}Z_{11}&Z_{12}&\cdots&Z_{1n}\\ Z_{21}&Z_{22}&\cdots&Z_{2n}\\ \vdots&\vdots&\cdots&\vdots\\ Z_{n1}&Z_{n2}&\cdots&Z_{nn}\end{bmatrix}\begin{bmatrix}\dot I_1\\ \dot I_2\\ \vdots\\ \dot I_n\end{bmatrix}$$

- 支路追加法生成Z阵-追加连支（方法II）



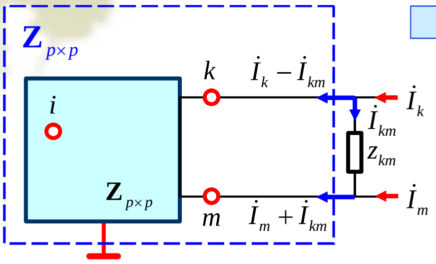
$$\dot{\mathbf{V}}=\mathbf{Z}'\dot{\mathbf{I}}$$

$$\dot V_i=\sum_{j=1}^{p}Z'_{ij}\dot I_j=Z'_{i1}\dot I_1+\cdots+Z'_{ik}\dot I_k+\cdots+Z'_{im}\dot I_m+\cdots+Z'_{ip}\dot I_p$$

4-3 节点阻抗矩阵

$$\begin{bmatrix}\dot V_1\\ \dot V_2\\ \vdots\\ \dot V_n\end{bmatrix}=\begin{bmatrix}Z_{11} & Z_{12} & \cdots & Z_{1n}\\ Z_{21} & Z_{22} & \cdots & Z_{2n}\\ \vdots & \vdots & \cdots & \vdots\\ Z_{n1} & Z_{n2} & \cdots & Z_{nn}\end{bmatrix}\begin{bmatrix}\dot I_1\\ \dot I_2\\ \vdots\\ \dot I_n\end{bmatrix}$$

- 支路追加法生成Z阵-追加连支（方法II）



$$\dot{\mathbf{V}}=\mathbf{Z}\dot{\mathbf{I}}'$$

$$\dot I'_k=\dot I_k-\dot I_{km}$$
$$\dot I'_m=\dot I_m+\dot I_{km}$$
$$\dot I'_j=\dot I_j,\ j\neq k,m$$

$$\dot V_i=\sum_{j=1}^{p}Z_{ij}\dot I'_j=Z_{i1}\dot I_1+\cdots+Z_{ik}\left(\dot I_k-\dot I_{km}\right)+\cdots+Z_{im}\left(\dot I_m+\dot I_{km}\right)+\cdots+Z_{ip}\dot I_p$$

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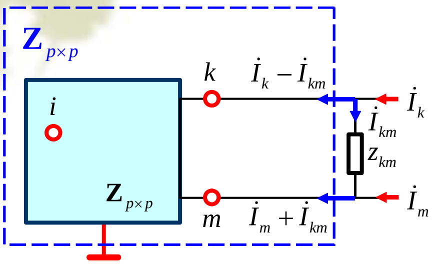
4-3 节点阻抗矩阵

$$\begin{bmatrix}\dot V_1\\ \dot V_2\\ \vdots\\ \dot V_n\end{bmatrix}=\begin{bmatrix}Z_{11} & Z_{12} & \cdots & Z_{1n}\\ Z_{21} & Z_{22} & \cdots & Z_{2n}\\ \vdots & \vdots & \cdots & \vdots\\ Z_{n1} & Z_{n2} & \cdots & Z_{nn}\end{bmatrix}\begin{bmatrix}\dot I_1\\ \dot I_2\\ \vdots\\ \dot I_n\end{bmatrix}$$

- 支路追加法生成Z阵-追加连支（方法II）



关键：$\dot I_{km}=?$

$$\dot V_i=\sum_{j=1}^{p}Z_{ij}\dot I_j-\left(Z_{ik}-Z_{im}\right)\dot I_{km}$$

$$\dot V_i=\sum_{j=1}^{p}Z_{ij}\dot I'_j=Z_{i1}\dot I_1+\cdots+Z_{ik}\left(\dot I_k-\dot I_{km}\right)+\cdots+Z_{im}\left(\dot I_m+\dot I_{km}\right)+\cdots+Z_{ip}\dot I_p$$

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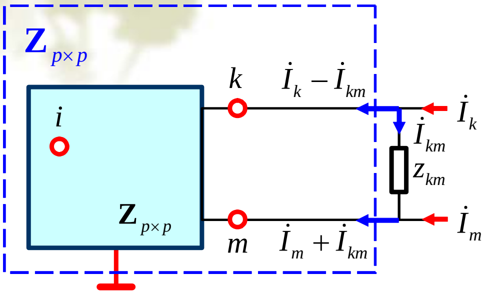
4-3 节点阻抗矩阵

$$\begin{bmatrix}\dot V_1\\ \dot V_2\\ \vdots\\ \dot V_n\end{bmatrix}=\begin{bmatrix}Z_{11} & Z_{12} & \cdots & Z_{1n}\\ Z_{21} & Z_{22} & \cdots & Z_{2n}\\ \vdots & \vdots & \cdots & \vdots\\ Z_{n1} & Z_{n2} & \cdots & Z_{nn}\end{bmatrix}\begin{bmatrix}\dot I_1\\ \dot I_2\\ \vdots\\ \dot I_n\end{bmatrix}$$

□ 支路追加法生成Z阵-追加连支（方法II）



$$\dot V_k = \sum_{j=1}^{p} Z_{kj}\dot I_j - (Z_{kk} - Z_{km})\dot I_{km}$$

$$\dot V_m = \sum_{j=1}^{p} Z_{mj}\dot I_j - (Z_{mk} - Z_{mm})\dot I_{km}$$

If: $i = k$ or m

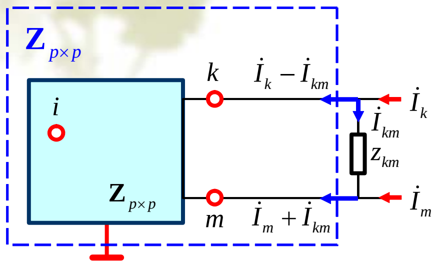
$$\dot V_i = \sum_{j=1}^{p} Z_{ij}\dot I_j - (Z_{ik} - Z_{im})\dot I_{km}$$

$$\dot V_i = \sum_{j=1}^{p} Z_{ij}\dot I'_j = Z_{i1}\dot I_1 + \cdots + Z_{ik}\left(\dot I_k - \dot I_{km}\right) + \cdots + Z_{im}\left(\dot I_m + \dot I_{km}\right) + \cdots + Z_{ip}\dot I_p$$

4-3 节点阻抗矩阵

$$\begin{bmatrix}\dot V_1\\ \dot V_2\\ \vdots\\ \dot V_n\end{bmatrix}=\begin{bmatrix}Z_{11} & Z_{12} & \cdots & Z_{1n}\\ Z_{21} & Z_{22} & \cdots & Z_{2n}\\ \vdots & \vdots & \cdots & \vdots\\ Z_{n1} & Z_{n2} & \cdots & Z_{nn}\end{bmatrix}\begin{bmatrix}\dot I_1\\ \dot I_2\\ \vdots\\ \dot I_n\end{bmatrix}$$

□ 支路追加法生成Z阵-追加连支（方法II）



$$\dot V_k = \sum_{j=1}^{p} Z_{kj}\dot I_j - (Z_{kk} - Z_{km})\dot I_{km}$$

$$\dot V_m = \sum_{j=1}^{p} Z_{mj}\dot I_j - (Z_{mk} - Z_{mm})\dot I_{km}$$

$$\dot V_k - \dot V_m = \sum_{j=1}^{p}\left(Z_{kj} - Z_{mj}\right)\dot I_j - \left(Z_{kk} - Z_{km} - Z_{mk} + Z_{mm}\right)\dot I_{km} = z_{km}\dot I_{km}$$

$$\dot I_{km} = \frac{1}{\left(Z_{kk} + Z_{mm} - 2Z_{km} + z_{km}\right)}\sum_{j=1}^{p}\left(Z_{kj} - Z_{mj}\right)\dot I_j$$

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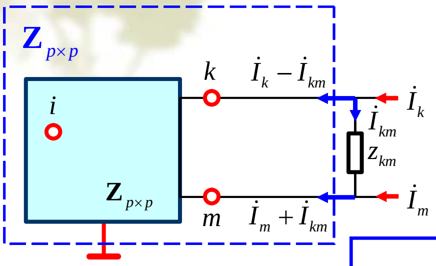
4-3 节点阻抗矩阵

$$\begin{bmatrix}\dot V_1\\ \dot V_2\\ \vdots\\ \dot V_n\end{bmatrix}=\begin{bmatrix}Z_{11} & Z_{12} & \cdots & Z_{1n}\\ Z_{21} & Z_{22} & \cdots & Z_{2n}\\ \vdots & \vdots & \cdots & \vdots\\ Z_{n1} & Z_{n2} & \cdots & Z_{nn}\end{bmatrix}\begin{bmatrix}\dot I_1\\ \dot I_2\\ \vdots\\ \dot I_n\end{bmatrix}$$

□ 支路追加法生成Z阵-追加连支（方法II）



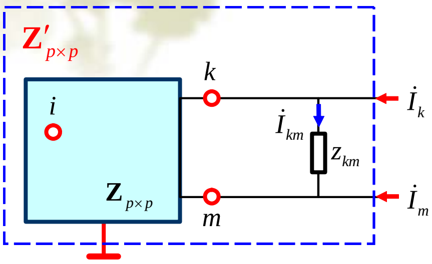
$$\dot V_i=\sum_{j=1}^{p}Z_{ij}\dot I_j-\left(Z_{ik}-Z_{im}\right)\dot I_{km}$$

$$\dot I_{km}=\frac{1}{\left(Z_{kk}+Z_{mm}-2Z_{km}+z_{km}\right)}\sum_{j=1}^{p}\left(Z_{kj}-Z_{mj}\right)\dot I_j$$

$$\dot V_i=\sum_{j=1}^{p}Z_{ij}\dot I_j-\frac{\left(Z_{ik}-Z_{im}\right)}{\left(Z_{kk}+Z_{mm}-2Z_{km}+z_{km}\right)}\sum_{j=1}^{p}\left(Z_{kj}-Z_{mj}\right)\dot I_j$$

4-3 节点阻抗矩阵

□ 支路追加法生成Z阵-追加连支（方法II）



$$Z'_{ij}=Z_{ij}-\frac{\left(Z_{ik}-Z_{im}\right)\left(Z_{kj}-Z_{mj}\right)}{\left(Z_{kk}+Z_{mm}-2Z_{km}+z_{km}\right)}$$
$$\left(i,j=1,2,\cdots,p\right)$$

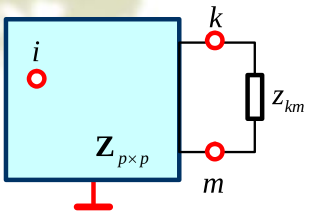
$$\dot V_i=\sum_{j=1}^{p}Z_{ij}\dot I_j-\frac{\left(Z_{ik}-Z_{im}\right)}{\left(Z_{kk}+Z_{mm}-2Z_{km}+z_{km}\right)}\sum_{j=1}^{p}\left(Z_{kj}-Z_{mj}\right)\dot I_j$$

$$\dot V_i=\sum_{j=1}^{p}Z'_{ij}\dot I_j$$

$$\dot V_i=\sum_{j=1}^{p}\left[Z_{ij}-\frac{\left(Z_{ik}-Z_{im}\right)\left(Z_{kj}-Z_{mj}\right)}{\left(Z_{kk}+Z_{mm}-2Z_{km}+z_{km}\right)}\right]\dot I_j$$

4-3 节点阻抗矩阵

□ 支路追加法生成Z阵-追加连支



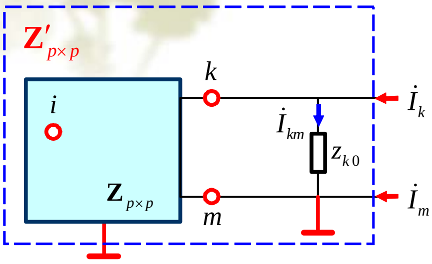
$\mathbf{Z}_{p\times p} \Rightarrow \mathbf{Z}'_{p\times p}$

$$\left[\begin{array}{ccccc|c} Z'_{11} & \cdots & Z'_{1m} & \cdots & Z'_{1k} & \cdots \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z'_{m1} & \cdots & Z'_{mm} & \cdots & Z'_{mk} & \cdots \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z'_{k1} & \cdots & Z'_{km} & \cdots & Z'_{kk} & \cdots \\ \hline \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \end{array}\right]$$

$$Z'_{ij} = Z_{ij} - \frac{\left(Z_{ik} - Z_{im}\right)\left(Z_{kj} - Z_{mj}\right)}{\left(Z_{kk} + Z_{mm} - 2Z_{km} + z_{km}\right)} \qquad (i, j = 1, 2, \cdots, p)$$

4-3 节点阻抗矩阵

□ 支路追加法生成Z阵-追加接地连支



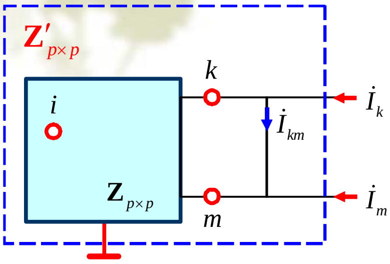
$$Z'_{ij} = Z_{ij} - \frac{\left(Z_{ik} - Z_{im}\right)\left(Z_{kj} - Z_{mj}\right)}{\left(Z_{kk} + Z_{mm} - 2Z_{km} + z_{km}\right)} \qquad (i, j = 1, 2, \cdots, p)$$

$$Z_{im} = Z_{mj} = Z_{km} = Z_{mm} = 0$$

$$Z'_{ij} = Z_{ij} - \frac{Z_{ik} Z_{kj}}{Z_{kk} + z_{k0}}, \quad (i, j = 1, 2, \cdots, p)$$

4-3 节点阻抗矩阵

□ 支路追加法生成Z阵-追加零阻抗连支



$$Z'_{ij} = Z_{ij} - \frac{(Z_{ik}-Z_{im})(Z_{kj}-Z_{mj})}{(Z_{kk}+Z_{mm}-2Z_{km}+z_{km})}$$

$$Z'_{ik} = Z_{ik} - \frac{(Z_{ik}-Z_{im})(Z_{kk}-Z_{mk})}{(Z_{kk}+Z_{mm}-2Z_{km}+z_{km})}$$

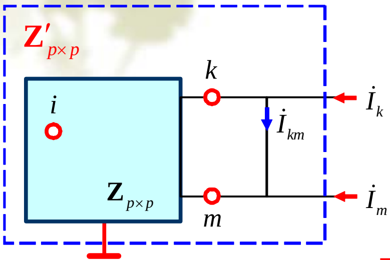
$$Z'_{im} = Z_{im} - \frac{(Z_{ik}-Z_{im})(Z_{km}-Z_{mm})}{(Z_{kk}+Z_{mm}-2Z_{km}+z_{km})}$$

$$Z'_{ik} - Z'_{im} = Z_{ik} - Z_{im} - \frac{(Z_{ik}-Z_{im})(Z_{kk}-Z_{mk})}{(Z_{kk}+Z_{mm}-2Z_{km}+z_{km})} + \frac{(Z_{ik}-Z_{im})(Z_{km}-Z_{mm})}{(Z_{kk}+Z_{mm}-2Z_{km}+z_{km})}$$

$$Z'_{ik} - Z'_{im} = (Z_{ik}-Z_{im})\left[1 - \frac{(Z_{kk}+Z_{mm}-2Z_{km})}{(Z_{kk}+Z_{mm}-2Z_{km}+z_{km})}\right]$$

4-3 节点阻抗矩阵

□ 支路追加法生成Z阵-追加零阻抗连支



$z_{km}=0$，相当于k、m节点合并，则有$Z'_{ik}-Z'_{im}=0$，即第k列和第m列元素完全相等

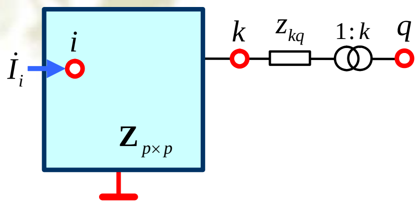
$$Z'_{ik} - Z'_{im} = (Z_{ik}-Z_{im})\left[1 - \frac{(Z_{kk}+Z_{mm}-2Z_{km})}{(Z_{kk}+Z_{mm}-2Z_{km}+0)}\right] = 0$$

$$Z'_{ik} - Z'_{im} = (Z_{ik}-Z_{im})\left[1 - \frac{(Z_{kk}-Z_{mk})-(Z_{km}-Z_{mm})}{(Z_{kk}+Z_{mm}-2Z_{km}+z_{km})}\right]$$

$z_{km}=0$

4-3 节点阻抗矩阵

□ 支路追加法生成Z阵-追加变压器树支



$$\left[\begin{array}{ccccc|c} Z_{11} & \cdots & Z_{1k} & \cdots & Z_{1p} & Z_{1q} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{k1} & \cdots & Z_{kk} & \cdots & Z_{kp} & Z_{kq} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pk} & \cdots & Z_{pp} & Z_{pq} \\ \hline Z_{q1} & \cdots & Z_{qk} & \cdots & Z_{qp} & Z_{qq} \end{array}\right]$$

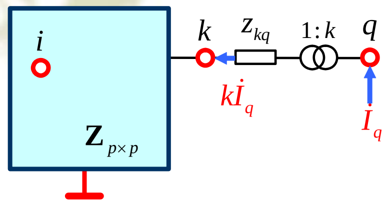
i节点单独注入电流$\dot I_i$

$\dot V_k = Z_{ki}\dot I_i$

$\dot V_q = Z_{qi}\dot I_i = k\dot V_k = kZ_{ki}\dot I_i$ ⇨ $Z_{qi} = kZ_{ki}$

4-3 节点阻抗矩阵

□ 支路追加法生成Z阵-追加变压器树支



$$\left[\begin{array}{ccccc|c} Z_{11} & \cdots & Z_{1k} & \cdots & Z_{1p} & Z_{1q} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{k1} & \cdots & Z_{kk} & \cdots & Z_{kp} & Z_{kq} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pk} & \cdots & Z_{pp} & Z_{pq} \\ \hline Z_{q1} & \cdots & Z_{qk} & \cdots & Z_{qp} & Z_{qq} \end{array}\right]$$

q节点单独注入电流$\dot I_q$

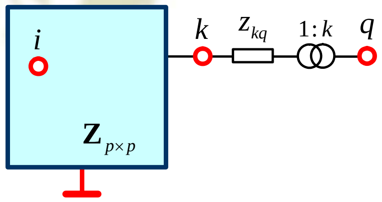
$\dot V_i = Z_{iq}\dot I_q = kZ_{ik}\dot I_q$ ⇨ $Z_{iq} = kZ_{ik}$

$\dot V_q = k\left(\dot V_k + z_{kq}k\dot I_q\right) = k^2\left(Z_{kk} + z_{kq}\right)\dot I_q = Z_{qq}\dot I_q$

$Z_{qq} = k^2\left(Z_{kk} + z_{kq}\right)$

4-3 节点阻抗矩阵

□ 支路追加法生成Z阵-追加变压器树支



$$\begin{bmatrix} Z_{11} & \cdots & Z_{1k} & \cdots & Z_{1p} & Z_{1q} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{k1} & \cdots & Z_{kk} & \cdots & Z_{kp} & Z_{kq} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pk} & \cdots & Z_{pp} & Z_{pq} \\ Z_{q1} & \cdots & Z_{qk} & \cdots & Z_{qp} & Z_{qq} \end{bmatrix}$$

Z阵增加一行一列

Z阵原有元素不变

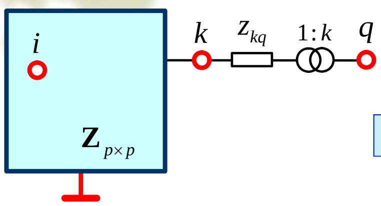
$$Z_{iq} = Z_{qi} = kZ_{ki}, (i = 1, 2, \cdots, p)$$

$$Z_{qq} = k^2 \left(Z_{kk} + z_{kq} \right)$$

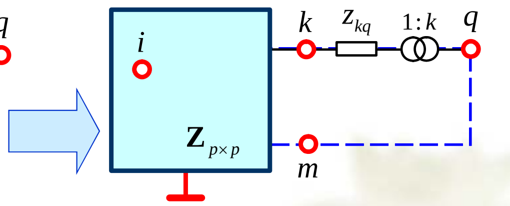
4-3 节点阻抗矩阵

□ 支路追加法生成Z阵-追加变压器连支

步骤一:



步骤二:



$\mathbf{Z}_{p\times p}$ ⇒ $\mathbf{Z}_{(p+1)\times(p+1)}$

$\mathbf{Z}'_{p\times p}$ ⇐ $\mathbf{Z}'_{(p+1)\times(p+1)}$

$$Z_{iq} = Z_{qi} = kZ_{ik}$$

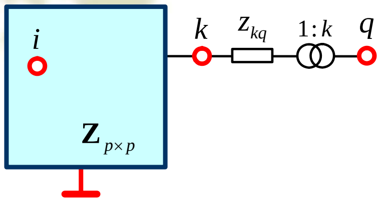
$$Z_{qq} = k^2 \left(Z_{kk} + z_{kq} \right)$$

$$Z'_{ij} = Z_{ij} - \frac{\left(Z_{iq} - Z_{im} \right)\left(Z_{qj} - Z_{mj} \right)}{\left(Z_{qq} + Z_{mm} - 2Z_{qm} \right)}$$

4-3 节点阻抗矩阵

□ 支路追加法生成Z阵-追加变压器连支

步骤一：



$\mathbf{Z}_{p\times p} \Rightarrow \mathbf{Z}_{(p+1)\times(p+1)}$

$$Z_{iq} = Z_{qi} = kZ_{ik}$$

$$Z_{qq} = k^2\left(Z_{kk} + z_{kq}\right)$$

$$\begin{bmatrix} Z_{11} & \cdots & Z_{1k} & \cdots & Z_{1p} & Z_{1q} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{k1} & \cdots & Z_{kk} & \cdots & Z_{kp} & Z_{kq} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pk} & \cdots & Z_{pp} & Z_{pq} \\ Z_{q1} & \cdots & Z_{qk} & \cdots & Z_{qp} & Z_{qq} \end{bmatrix}$$

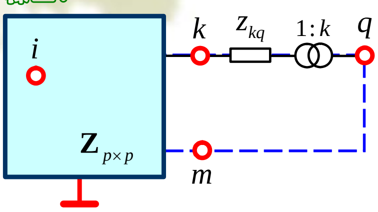
Z阵增加一行一列

Z阵原有元素不变

4-3 节点阻抗矩阵

□ 支路追加法生成Z阵-追加变压器连支

步骤二：



$$Z'_{ij} = Z_{ij} - \frac{\left(Z_{iq} - Z_{im}\right)\left(Z_{qj} - Z_{mj}\right)}{\left(Z_{qq} + Z_{mm} - 2Z_{qm}\right)}$$

代入

$$Z_{iq} = Z_{qi} = kZ_{ik}, Z_{qq} = k^2\left(Z_{kk} + z_{kq}\right)$$

$\mathbf{Z}'_{p\times p}$

$$Z'_{ij} = Z_{ij} - \frac{\left(kZ_{ik} - Z_{im}\right)\left(kZ_{kj} - Z_{mj}\right)}{k^2\left(Z_{kk} + z_{kq}\right) + Z_{mm} - 2kZ_{mk}}, (i, j = 1, 2, \cdots, p)$$

4-3 节点阻抗矩阵

□ 由线性代数方程YZ=I计算Z阵

$$\begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$\mathbf{Y}[\mathbf{Z}_1\ \mathbf{Z}_2\ \cdots\ \mathbf{Z}_n] = [\mathbf{e}_1\ \mathbf{e}_2\ \cdots \mathbf{e}_n]$$

$$\mathbf{Z}_j = [Z_{1j}\ Z_{2j}\ \cdots\ Z_{nj}]^{\mathrm{T}}$$

$$\mathbf{YZ}_j = \mathbf{e}_j, (j = 1, 2, \cdots, n)$$

$$\mathbf{e}_j = [0\ \cdots\ 1\ \cdots\ 0]^{\mathrm{T}}$$

第j列

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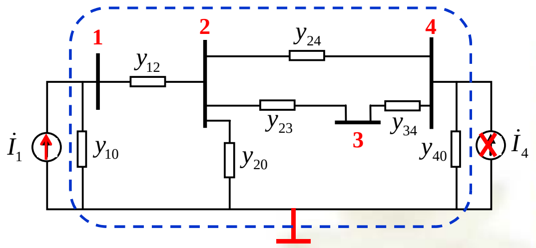
4-3 节点阻抗矩阵

□ 由线性代数方程YZ=I计算Z阵

$$\begin{bmatrix} Y_{11} & \cdots & Y_{1j} & \cdots & Y_{1n} \\ \vdots & & \vdots & & \vdots \\ Y_{j1} & \cdots & Y_{jj} & \cdots & Y_{jn} \\ \vdots & & \vdots & & \vdots \\ Y_{n1} & \cdots & Y_{nj} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} Z_{1j} \\ \vdots \\ Z_{jj} \\ \vdots \\ Z_{nj} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$



$$\mathbf{YZ}_j = \mathbf{e}_j$$

$$\mathbf{Y}\dot{\mathbf{V}} = \dot{\mathbf{I}}$$

$\dot I_i = 0,$
$\dot I_j = 1,$
$(i \neq j)$

$$\dot V_j = Z_{jj}\dot I_j = Z_{jj}, \quad \dot V_i = Z_{ij}\dot I_j = Z_{ij}$$

$YZ_j = e_j$ 方程的物理意义：e_j作为节点注入电流列相量，Z_j就是节点电压列相量。

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4-3 节点阻抗矩阵

- 由线性代数方程YZ=I计算Z阵

$$\begin{bmatrix} Y_{11} & \cdots & Y_{1j} & \cdots & Y_{1n} \\ \vdots & & \vdots & & \vdots \\ Y_{j1} & \cdots & Y_{jj} & \cdots & Y_{jn} \\ \vdots & & \vdots & & \vdots \\ Y_{n1} & \cdots & Y_{nj} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} Z_{1j} \\ \vdots \\ Z_{jj} \\ \vdots \\ Z_{nj} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$

$$\mathbf{YZ}_j = \mathbf{e}_j \qquad \mathbf{LDUZ}_j = \mathbf{e}_j$$

$$\mathbf{Y} = \mathbf{LDU}$$

$$\begin{bmatrix} Y_{11} & \cdots & Y_{1j} & \cdots & Y_{1n} \\ \vdots & & \vdots & & \vdots \\ Y_{j1} & \cdots & Y_{jj} & \cdots & Y_{jn} \\ \vdots & & \vdots & & \vdots \\ Y_{n1} & \cdots & Y_{nj} & \cdots & Y_{nn} \end{bmatrix} = \begin{bmatrix} 1 & & & & \\ l_{21} & 1 & & & \\ l_{31} & l_{32} & 1 & & \\ \vdots & \vdots & \vdots & \ddots & \\ l_{n1} & l_{n2} & l_{n3} & \cdots & 1 \end{bmatrix} \begin{bmatrix} d_{11} & & & & \\ & d_{22} & & & \\ & & d_{33} & & \\ & & & \ddots & \\ & & & & d_{nn} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} & \cdots & u_{1n} \\ & 1 & u_{23} & \cdots & u_{2n} \\ & & 1 & \cdots & u_{3n} \\ & & & \ddots & \vdots \\ & & & & 1 \end{bmatrix}$$

4-3 节点阻抗矩阵

- 由线性代数方程YZ=I计算Z阵

$$\mathbf{LDUZ}_j = \mathbf{e}_j$$

$$\mathbf{LF} = \mathbf{e}_j$$

$$f_i = \begin{cases} 0 & i < j \\ 1 & i = j \\ -\sum_{k=j}^{i-1} l_{ik} f_k, & i > j \end{cases}$$

$$\begin{bmatrix} 1 & & & & & & & & \\ l_{21} & 1 & & & & & & & \\ l_{31} & l_{32} & 1 & & & & & & \\ \vdots & \vdots & \vdots & \ddots & & & & & \\ l_{j1} & l_{j2} & l_{j3} & \cdots & 1 & & & & \\ \vdots & \vdots & \vdots & & \vdots & \ddots & & & \\ l_{i1} & l_{i2} & l_{i3} & \cdots & l_{ij} & \cdots & 1 & & \\ \vdots & \vdots & \vdots & & \vdots & & \vdots & \ddots & \\ l_{n1} & l_{n2} & l_{n3} & \cdots & l_{nj} & \cdots & l_{ni} & \cdots & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_j \\ \vdots \\ f_i \\ \vdots \\ f_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

4-3 节点阻抗矩阵

- 由线性代数方程YZ=I计算Z阵

$\mathbf{LDUZ}_j = \mathbf{e}_j$

$\mathbf{DUZ}_j = \mathbf{F}$

$\mathbf{DH} = \mathbf{F}$

$$\begin{bmatrix} d_{11} & & & & & & & & \\ & d_{22} & & & & & & & \\ & & d_{33} & & & & & & \\ & & & \ddots & & & & & \\ & & & & d_{jj} & & & & \\ & & & & & \ddots & & & \\ & & & & & & d_{ii} & & \\ & & & & & & & \ddots & \\ & & & & & & & & d_{nn} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ \vdots \\ h_j \\ \vdots \\ h_i \\ \vdots \\ h_n \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_j \\ \vdots \\ f_i \\ \vdots \\ f_n \end{bmatrix}$$

$$h_i = \begin{cases} 0 & i < j \\ f_i / d_{ii} & i \ge j \end{cases}$$

4-3 节点阻抗矩阵

- 由线性代数方程YZ=I计算Z阵

$\mathbf{LDUZ}_j = \mathbf{e}_j$

$\mathbf{LF} = \mathbf{e}_j$

$\mathbf{DUZ}_j = \mathbf{F}$

$\mathbf{DH} = \mathbf{F}$

$\mathbf{UZ}_j = \mathbf{H}$

$$\begin{bmatrix} 1 & u_{12} & u_{13} & \cdots & u_{1j} & \cdots & u_{1i} & \cdots & u_{1n} \\ & 1 & u_{23} & \cdots & u_{2j} & \cdots & u_{2i} & \cdots & u_{2n} \\ & & 1 & \cdots & u_{3j} & \cdots & u_{3i} & \cdots & u_{3n} \\ & & & \ddots & \vdots & & \vdots & & \vdots \\ & & & & 1 & & u_{ji} & \cdots & u_{jn} \\ & & & & & \ddots & \vdots & & \vdots \\ & & & & & & 1 & \cdots & u_{in} \\ & & & & & & & \ddots & \vdots \\ & & & & & & & & 1 \end{bmatrix} \begin{bmatrix} Z_{1j} \\ Z_{2j} \\ Z_{3j} \\ \vdots \\ Z_{jj} \\ \vdots \\ Z_{ij} \\ \vdots \\ Z_{nj} \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ \vdots \\ h_j \\ \vdots \\ h_i \\ \vdots \\ h_n \end{bmatrix}$$

$$Z_{ij} = h_i - \sum_{k=i+1}^{n} u_{ik} Z_{kj}, \quad i = n, n-1, \cdots, 1$$

4-3 节点阻抗矩阵

- 由线性代数方程YZ=I计算Z阵-因子表

$$\mathbf{L}^{\mathrm{T}} = \mathbf{U}$$

$$l_{ki} = u_{ik}$$

$$\begin{bmatrix}
d_{11}^{-1} & u_{12} & u_{13} & \cdots & u_{1j} & \cdots & u_{1i} & \cdots & u_{1n} \\
 & d_{22}^{-1} & u_{23} & \cdots & u_{2j} & \cdots & u_{2i} & \cdots & u_{2n} \\
 & & d_{33}^{-1} & \cdots & u_{3j} & \cdots & u_{3i} & \cdots & u_{3n} \\
 & & & \ddots & \vdots & & \vdots & & \vdots \\
 & & & & d_{jj}^{-1} & & u_{ji} & \cdots & u_{jn} \\
 & & & & & \ddots & \vdots & & \vdots \\
 & & & & & & d_{ii}^{-1} & \cdots & u_{in} \\
 & & & & & & & \ddots & \vdots \\
 & & & & & & & & d_{nn}^{-1}
\end{bmatrix}$$

4-4 节点编号顺序的优化

节点编号顺序优化的目的是使得节点导纳阵在三角分解过程中尽可能地保持稀疏性，减少非零注入元，以节约内存和计算时间。

原则： 消去时增加新支路最少的节点应该优先编号。

简化： 按节点的连接支路数（接地支路除外）最少进行编号。（静态编号、动态编号）

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本章小结

- Y阵元素的物理意义；Y阵的特点、形成和修改。
- Z阵元素的物理意义，根据Z阵元素的物理意义形成Z阵的方法。
- 利用线性代数方程$YZ_j=e_j$计算Z阵的某一列元素。

如何利用学过的知识（电路理论、线性代数）将复杂问题进行分解和转化？

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习 题

Ex 4-1，4-2，4-4